Nonlinear Analysis Forum **13**(2), pp. 215–216, 2008
Available electronically at http://www.na-forum.org

A NOTE ON "APPROXIMATING FIXED POINTS OF HOLOMORPHIC MAPPINGS IN THE HILBERT BALL"

Arif Rafiq

Department of Mathematics
COMSATS Institute of Information Technology
Lahore, Pakistan
E-mail : arafiq@comsats.edu.pk

Abstract. In this note, we point out some mistakes appeared in [1].

Introduction and comments

In [1], Levenshtein and Reich proved the following theorem.

Theorem 1. *Let* $T \in Hol(B)$ *with* $F(T) \neq \varphi$, $f : B \to \alpha B$ *a holomorphic mapping, where* $0 \leq \alpha_n \leq 1$, $\{\alpha_n : n \in \mathbb{N}\}$ *a sequence satisfying* (i) $\lim\limits_{n\to\infty} \alpha_n = 1$, (ii) $\sum_{n=1}^{\infty} = \infty$, (iii) $\lim\limits_{n\to\infty} \frac{\alpha_n - \alpha_{n-1}}{(1-\alpha_n)^2} = 0$, *and* z_0 *a point in* B. *Then the sequence* $\{z_n : n \in \mathbb{N}\}$ *defined by*

$$z_n = (1-\alpha_n) f(z_{n-1}) + \alpha_n T z_{n-1},$$

converges strongly to the unique solution $v \in B$ *of the equation* $z = Q_{F(T)}(f(z))$, *where* $Q_{F(T)} : B \longmapsto F(T)$ *is the unique (holomorphic) retraction of* B *onto* $F(T)$ *which is firmly nonexpansive of the second kind.*

Remark 2. ([1]) Conditions (i), (ii), and (iii) are satisfied, for instance, when for each $n \in \mathbb{N}$, $\alpha_n = 1 - \frac{1}{n^\beta}$ with $0 < \beta < 1$.

We comment as follows:

- For $\alpha_n = 1 - \frac{1}{n}$, $\lim\limits_{n\to\infty} \frac{\alpha_n - \alpha_{n-1}}{(1-\alpha_n)^2} = 1$.
- The condition $\lim\limits_{n\to\infty} \frac{\alpha_n - \alpha_{n-1}}{(1-\alpha_n)^2} = 0$ is unnecessary, one can easily see that it is deduced from $\lim\limits_{n\to\infty} \alpha_n = 1$. Indeed

$$\frac{\alpha_n - \alpha_{n-1}}{(1-\alpha_n)^2} = \alpha_n + 2\alpha_n^2 + 3\alpha_n^3 + ... + k\alpha_n^k + ...$$
$$- \alpha_{n-1}(1 + 2\alpha_n + 3\alpha_n^2 + 4\alpha_n^3 + ... + (k+1)\alpha_n^k + ...).$$

2000 Mathematics Subject Classification: Primary 05C38, 15A15; Secondary 05A15, 15A18.
Key words and phrases: Approximating curve, Firmly nonexpansive mapping, Fixed point, Hilbert ball, Holomorphic mapping, Hyperbolic metric, Iterative scheme, Retraction, Sink point.

References

[1] M. Levenshtein, S. Reich, Approximating fixed points of holomorphic mappings in the Hilbert ball, Nonlinear Analysis (2008), doi:10.1016/j.na.2008.09.00.